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Geometric and Algebraic Topological Methods in Quantum Mechanics

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Preface

Contemporary quantum mechanics meets an explosion of different types of quantization. Some of these quantization techniques (geometric quantization, deformation quantization, BRST quantization, noncommutative geometry, quantum groups, etc.) call into play advanced geometry and algebraic topology. These techniques possess the following main peculiarities.

- Quantum theory deals with infinite-dimensional manifolds and fibre bundles as a rule.
- Geometry in quantum theory speaks mainly the algebraic language of rings, modules, sheaves and categories.
- Geometric and algebraic topological methods can lead to non-equivalent quantizations of a classical system corresponding to different values of topological invariants.

Geometry and topology are by no means the primary scope of our book, but they provide the most effective contemporary schemes of quantization. At the same time, we present in a compact way all the necessary up to date mathematical tools to be used in studying quantum problems.

Our book addresses to a wide audience of theoreticians and mathematicians, and aims to be a guide to advanced geometric and algebraic topological methods in quantum theory. Leading the reader to these frontiers, we hope to show that geometry and topology underlie many ideas in modern quantum physics. The interested reader is referred to extensive Bibliography spanning mostly the last decade. Many references we quote are duplicated in *E-print arXiv* (<http://xxx.lanl.gov>).

With respect to mathematical prerequisites, the reader is expected to be familiar with the basics of differential geometry of fibre bundles. For the sake of convenience, a few relevant mathematical topics are compiled in Appendixes.

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